

Package: CopulaInference (via r-universe)

October 8, 2024

Type Package

Title Estimation and Goodness-of-Fit of Copula-Based Models with Arbitrary Distributions

Version 0.5.0

Description Estimation and goodness-of-fit functions for copula-based models of bivariate data with arbitrary distributions (discrete, continuous, mixture of both types). The copula families considered here are the Gaussian, Student, Clayton, Frank, Gumbel, Joe, Plackett, BB1, BB6, BB7, BB8, together with the following non-central squared copula families in Nasri (2020) <[doi:10.1016/j.spl.2020.108704](https://doi.org/10.1016/j.spl.2020.108704)>: ncs-gaussian, ncs-clayton, ncs-gumbel, ncs-frank, ncs-joe, and ncs-plackett. For theoretical details, see, e.g., Nasri and Remillard (2023) <[arXiv:2301.13408](https://arxiv.org/abs/2301.13408)>.

License GPL-3

Encoding UTF-8

LazyData true

Depends R (>= 3.5.0), doParallel, parallel, foreach, stats, rvinecopulib, Matrix

RoxygenNote 7.2.3

NeedsCompilation yes

Author Bouchra R. Nasri [aut, cre, cph], Bruno N Remillard [aut]

Maintainer Bouchra R. Nasri <bouchra.nasri@umontreal.ca>

Date/Publication 2023-04-21 07:32:37 UTC

Repository <https://bouchranasri.r-universe.dev>

RemoteUrl <https://github.com/cran/CopulaInference>

RemoteRef HEAD

RemoteSha f107bafbcfc00c08180584b1245224f914be11a5

Contents

AuxFun	2
AuxFunC	3
BiEmpCdf	4
CdfInv	5
dncs	5
dplac	6
EmpCdf	7
EstBiCop	7
EstDep	9
est_options	10
Finv	11
fnumber	11
GofBiCop	12
hnCS	14
hplac	15
identifiability	15
pncs	16
pplac	17
preparedata	18
rhoplackett	18
rncs	19
rplac	20
simgumbel	20
statcvm	21
taucop	22
tauplackett	22
Index	24

AuxFun	<i>Auxiliary functions</i>
--------	----------------------------

Description

This function computes the empirical margins, their left-limits, Kendall's tau and Spearman's rho for arbitrary data. Slower than AuxFunC based on C.

Usage

```
AuxFun(data)
```

Arguments

data	Matrix (x,y) of size n x 2
------	----------------------------

Value

tau	Kendall's tau
rho	Spearman's rho
Fx	Empirical cdf of x
Fxm	Left-limit of the empirical cdf of x
Fy	Empirical cdf of y
Fym	Left-limit of the empirical cdf of y

References

Nasri (2022). Test of serial dependence for arbitrary distributions. *JMVA*

Nasri & Remillard (2023). Tests of independence and randomness for arbitrary data using copula-based covariances, arXiv 2301.07267.

Examples

```
data(simumbel)
out=AuxFun(simumbel)
```

 AuxFunC

Auxiliary functions using C

Description

This function computes the empirical margins, their left-limits, Kendall's tau and Spearman's rho for arbitrary data

Usage

```
AuxFunC(data)
```

Arguments

data	Matrix (x,y) of size n x 2
------	----------------------------

Value

tau	Kendall's tau
rho	Spearman's rho
Fx	Empirical cdf of x
Fxm	Left-limit of the empirical cdf of x
Fy	Empirical cdf of y
Fym	Left-limit of the empirical cdf of y

References

Nasri (2022). Test of serial dependence for arbitrary distributions. JMVA

Nasri & Remillard (2023). Tests of independence and randomness for arbitrary data using copula-based covariances, arXiv 2301.07267.

Examples

```
data(singumbel)
out=AuxFunc(singumbel)
```

BiEmpCdf

Empirical bivariate cdf

Description

This function computes the empirical joint cdf evaluated at all points (y1,y2)

Usage

```
BiEmpCdf(data, y1, y2)
```

Arguments

data	Matrix (x1,x2) of size n x 2
y1	Vector of size n1
y2	Vector of size n2

Value

cdf	Empirical cdf
-----	---------------

Examples

```
data(singumbel)
out=BiEmpCdf(singumbel,c(0,1),c(-1,0,1))
```

CdfInv	<i>Quantile function</i>
--------	--------------------------

Description

This function computes the inverse of the cdf of a finite distribution for a vector of probabilities.

Usage

```
CdfInv(u, y, Fn)
```

Arguments

u	Vector of probabilities
y	Ordered values
Fn	Cdf

Value

x	Vector of quantiles
---	---------------------

Examples

```
y=c(0,1,2)
Fn = c(0.5,0.85,1)
out=CdfInv(c(1:9)/10,y,Fn)
```

dncs	<i>Density of non-central squared copula</i>
------	--

Description

This function computes the density of the non-central squared copula (nsc) associated with a one-parameter copula with parameter cpar, and parameters a1, a2 >0 .

Usage

```
dncs(data, family, rotation = 0, par)
```

Arguments

data	Matrix (x,y) of size n x 2
family	Copula family: "ncs-gaussian", "ncs-clayton", "ncs-frank", "ncs-gumbel", "ncs-joe", "ncs-plackett".
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	vector of copula parameter and non-centrality parameter a1,a2 >0

Value

pdf	Density
-----	---------

References

Nasri (2020). On non-central squared copulas. *Statistics and Probability Letters*.

Examples

```
dncs(c(0.5,0.8),"ncs-clayton",par=c(2,1,2))
```

dplac

Density of Plackett copula

Description

This function computes the density of the Plackett copula with parameter $\text{par} > 0$.

Usage

```
dplac(data, rotation = 0, par)
```

Arguments

data	Matrix (x,y) of size n x 2
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	Copula parameter >0

Value

pdf	Density
-----	---------

Examples

```
dplac(c(0.5,0.8),par=3,rotation=270)
```

EmpCdf	<i>Empirical univariate cdf</i>
--------	---------------------------------

Description

This function computes the empirical cdf evaluated at all sample points

Usage

```
EmpCdf(x)
```

Arguments

x	Observations
---	--------------

Value

Fx	Empirical cdf
Fxm	Left limit of the empirical cdf
Ix	Indicator of atoms

<>

Examples

```
data(singumbel)
out=EmpCdf(singumbel[,1])
```

EstBiCop	<i>Parameter estimation for bivariate copula-based models with arbitrary distributions</i>
----------	--

Description

Computes the estimation of the parameters of a copula-based model with arbitrary distributions, i.e., possibly mixtures of discrete and continuous distributions. Parametric margins are allowed. The estimation is based on a pseudo-likelihood adapted to ties.

Usage

```
EstBiCop(
  data = NULL,
  family,
  rotation = 0,
  Fx = NULL,
  Fxm = NULL,
  Fy = NULL,
  Fym = NULL
)
```

Arguments

<code>data</code>	Matrix or data frame with 2 columns (X,Y). Can be pseudo-observations. If NULL, Fx and Fy must be provided.
<code>family</code>	Copula family: "gaussian", "t", "clayton", "frank", "gumbel", "joe", "plackett", "bb1", "bb6", "bb7", "bb8", "ncs-gaussian", "ncs-clayton", "ncs-gumbel", "ncs-frank", "ncs-joe", "ncs-plackett".
<code>rotation</code>	Rotation: 0 (default value), 90, 180, or 270.
<code>Fx</code>	Marginal cdf function applied to X (default is NULL).
<code>Fxm</code>	Left-limit of marginal cdf function applied to X default is NULL).
<code>Fy</code>	Marginal cdf function applied to Y (default is NULL).
<code>Fym</code>	Left-limit of marginal cdf function applied to Y (default is NULL).

Value

<code>par</code>	Copula parameters
<code>family</code>	Copula family
<code>rotation</code>	Rotation value
<code>tauth</code>	Kendall's tau corresponding to the estimated parameter
<code>tauemp</code>	Empirical Kendall's tau (from the multilinear empirical copula)
<code>rhoSth</code>	Spearman's rho corresponding to the estimated parameter
<code>rhoSemp</code>	Empirical Spearman's tau (from the multilinear empirical copula)
<code>loglik</code>	Log-likelihood
<code>aic</code>	Aic value
<code>bic</code>	Bic value
<code>data</code>	Matrix of values (could be (Fx,Fy))
<code>F1</code>	Cdf of X (Fx if provided, empirical otherwise)
<code>F1m</code>	Left-limit of F1 (Fxm if provided, empirical otherwise)
<code>F2</code>	Cdf of Y (Fy if provided, empirical otherwise)
<code>F2m</code>	Left-limit of F2 (Fym if provided, empirical otherwise)
<code>ccdfx</code>	Conditional cdf of X given Y and it left limit

ccdfxm	Left-limit of ccdfx
ccdfy	Conditional cdf of Y given X and it left limit
ccdfym	Left-limit of ccdfy

References

Nasri & Remillard (2023). Identifiability and inference for copula-based semiparametric models for random vectors with arbitrary marginal distributions. arXiv 2301.13408.

Nasri (2020). On non-central squared copulas. Statistics and Probability Letters.

Examples

```
set.seed(2)
data = matrix(rpois(20,1),ncol=2)
out0=EstBiCop(data,"gumbel")
```

EstDep	<i>Kendall's tau and Spearman's rho</i>
--------	---

Description

This function computes Kendall's tau and Spearman's rho for arbitrary data. These are invariant by increasing mappings.

Usage

```
EstDep(data)
```

Arguments

data	Matrix or data frame with 2 columns (X,Y). Can be pseudo-observations.
------	--

Value

tau	Kendall's tau
rho	Spearman's rho

References

Nasri (2022). Test of serial dependence for arbitrary distributions. JMVA

Nasri & Remillard (2023). Tests of independence and randomness for arbitrary data using copula-based covariances, arXiv 2301.07267.

Examples

```
data(singumbel)
out=EstDep(singumbel)
```

est_options	<i>Options for the estimation of the parameters of bivariate copula-based models</i>
-------------	--

Description

Sets starting values, upper and lower bounds for the parameters. The bounds are based on those in the rvinecopulib package.

Usage

```
est_options(family, tau = 0.5)
```

Arguments

family	Copula family: "gaussian", "t", "clayton", "frank", "gumbel", "joe", "plackett", "bb1", "bb6", "bb7", "bb8", "ncs-gaussian", "ncs-clayton", "ncs-gumbel", "ncs-frank", "ncs-joe", "ncs-plackett".
tau	Estimated Kendall's tau to compute a starting point (default is 0.5)

Value

LB	Lower bound for the parameters
UB	Upper bound for the parameters
start	Starting point for the estimation

References

Nagler & Vatter (2002). rvinecopulib: High Performance Algorithms for Vine Copula Modeling. Version 0.6.2.1.3

Nasri (2020). On non-central squared copulas. *Statistics and Probability Letters*.

Nasri (2022). Test of serial dependence for arbitrary distributions. *JMVA*.

Nasri & Remillard (2023). Copula-based dependence measures for arbitrary data, arXiv 2301.07267.

Examples

```
out = est_options("bb8")
```

Finv	<i>Quantile function of margins</i>
------	-------------------------------------

Description

This function computes the quantile of seven cdf used in Nasri (2022).

Usage

```
Finv(u, k)
```

Arguments

u	Vector of probabilities
k	Marginal distribution: [1] Bernoulli(0.8), [2] Poisson(6), [3] Negative binomial with $r = 1.5$, $p = 0.2$, [4] Zero-inflated Poisson (10) with $w = 0.1$ and $P(6.67)$ otherwise, [5] Zero-inflated Gaussian, [6] Discretized Gaussian, [7] Discrete Pareto(1)

Value

x	Vector of quantiles
---	---------------------

Author(s)

Bouchra R. Nasri January 2021

References

B.R Nasri (2022). Tests of serial dependence for arbitrary distributions

Examples

```
x = Finv(runif(40),2)
```

fnumber	<i>Family number corresponding to VineCopula package</i>
---------	--

Description

Computes the number associated with a copula family (without rotation)

Usage

```
fnumber(family)
```

Arguments

family Copula family: "gaussian", "t", "clayton", "frank", "gumbel", "joe", "plackett", "bb1", "bb6", "bb7", "bb8".

Value

fnumber Number

References

Nagler et al. (2023). VineCopula: Statistical Inference of Vine Copulas, version 2.4.5.

Examples

```
fnumber("bb1")
```

GofBiCop

Goodness-of-fit for bivariate copula-based models with arbitrary distributions

Description

Goodness-of-fit tests for copula-based models for data with arbitrary distributions. The tests statistics are the Cramer-von Mises statistic (S_n), the difference between the empirical Kendall's tau and the theoretical one, and the difference between the empirical Spearman's rho and the theoretical one.

Usage

```
GofBiCop(
  data = NULL,
  family,
  rotation = 0,
  Fx = NULL,
  Fxm = NULL,
  Fy = NULL,
  Fym = NULL,
  B = 100,
  n_cores = 1
)
```

Arguments

data	Matrix or data frame with 2 columns (X,Y). Can be pseudo-observations. If NULL, Fx and Fy must be provided.
family	Copula family: "gaussian", "t", "clayton", "frank", "gumbel", "joe", "plackett", "bb1", "bb6", "bb7", "bb8", "ncs-gaussian", "ncs-clayton", "ncs-gumbel", "ncs-frank", "ncs-joe", "ncs-plackett".
rotation	Rotation: 0 (default value), 90, 180, or 270.
Fx	marginal cdf function applied to X (default is NULL).
Fxm	left limit of marginal cdf function applied to X default is NULL).
Fy	marginal cdf function applied to Y (default is NULL).
Fym	left limit of marginal cdf function applied to Y (default is NULL).
B	Number of bootstrap samples (default 100)
n_cores	Number of cores to be used for parallel computing (default is 1).

Value

pvalueSn	Pvalue of Sn in percent
pvalueTn	Pvalue of Tn in percent
pvalueRn	Pvalue of Rn in percent
Sn	Value of Cramer-von Mises statistic Sn
Tn	Value of Kendall's statistic Tn
Rn	Value of Spearman's statistic Rn
cpar	Copula parameters
family	Copula family
rotation	Rotation value
tauth	Kendall's tau (from the multilinear theoretical copula)
tauemp	Empirical Kendall's tau (from the multilinear empirical copula)
rhoth	Spearman's rho (from the multilinear theoretical copula)
rhoemp	Empirical Spearman's rho (from the multilinear empirical copula)
parB	Bootstrapped parameters
loglik	Log-likelihood
aic	AIC value
bic	BIC value

References

- Nasri & Remillard (2023). Identifiability and inference for copula-based semiparametric models for random vectors with arbitrary marginal distributions. arXiv 2301.13408.
- Nasri & Remillard (2023). Goodness-of-fit and bootstrapping for copula-based random vectors with arbitrary marginal distributions.
- Nasri (2020). On non-central squared copulas. Statistics and Probability Letters.

Examples

```
data = rvinecopulib::rbicop(10,"gumbel",rotation=0,2)
out=GofBiCop(data,family="gumbel",B=10)
```

hncs

Conditional distribution of non-central squared copula

Description

This function computes the conditional distribution of the non-central squared copula (ncs) associated with a one-parameter copula with parameter cpar, and parameters a1, a2 >0 .

Usage

```
hncs(data, cond_var, family, rotation = 0, par)
```

Arguments

data	Matrix (x,y) of size n x 2
cond_var	Conditioning variable (1 or 2)
family	Copula family: "ncs-gaussian", "ncs-clayton", "ncs-frank", "ncs-gumbel", "ncs-joe", "ncs-plackett".
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	vector of copula parameter and non-centrality parameter a1,a2 >0

Value

h	Conditional cdf
---	-----------------

References

Nasri (2020). On non-central squared copulas. *Statistics and Probability Letters*.

Examples

```
hncs(c(0.5,0.8),1,"ncs-clayton",270,c(2,1,2))
```

hplac *Conditional distribution of Plackett copula*

Description

This function computes the conditional distribution of the Plackett copula with parameter $\text{par} > 0$.

Usage

```
hplac(data, cond_var, rotation = 0, par)
```

Arguments

data	Matrix (x,y) of size n x 2
cond_var	Conditioning variable (1 or 2)
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	Copula parameter > 0

Value

h	Conditional cdf
---	-----------------

Examples

```
hplac(c(0.5,0.8),1,270,3)
```

identifiability *Identifiability of two-parameter copula families*

Description

Determines if a copula family is identifiable with respect to the empirical margins. One-parameter copula families ("gaussian", "gumbel", "clayton", "frank", "plackett", "joe") are identifiable whatever the margins. The rank of the gradient of the copula on the range of the margins is evaluated at 10000 parameter points within the lower and upper bounds of the copula family.

Usage

```
identifiability(data = NULL, family, rotation = 0, Fx = NULL, Fy = NULL)
```

Arguments

data	Matrix or data frame with 2 columns (X,Y). Can be pseudo-observations. If NULL, Fx and Fy must be provided.
family	Copula family: "gaussian", "t", "clayton", "frank", "gumbel", "joe", "plackett", "bb1", "bb6", "bb7", "bb8", "ncs-gaussian", "ncs-clayton", "ncs-gumbel", "ncs-frank", "ncs-joe", "ncs-plackett".
rotation	Rotation: 0 (default value), 90, 180, or 270.
Fx	Marginal cdf function applied to X (default is NULL).
Fy	Marginal cdf function applied to Y (default is NULL).

Value

out	True or False
-----	---------------

References

Nasri & Remillard (2023). Identifiability and inference for copula-based semiparametric models for random vectors with arbitrary marginal distributions. arXiv 2301.13408.

Nasri (2020). On non-central squared copulas. Statistics and Probability Letters.

Examples

```
set.seed(1)
data = matrix(rpois(20,1),ncol=2)
out = identifiability(data,"gumbel")
```

pnsc

Cdf for non-central squared copula

Description

This function computes the distribution function of the non-central squared copula (ncs) associated a with one-parameter copula with parameter cpar, and parameters a1, a2 >0 .

Usage

```
pnsc(data, family, rotation = 0, par)
```

Arguments

data	Matrix (x,y) of size n x 2
family	Copula family: "ncs-gaussian", "ncs-clayton", "ncs-frank", "ncs-gumbel", "ncs-joe", "ncs-plackett".
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	vector of copula parameter and non-centrality parameter a1,a2 >0

Value

cdf	Value of cdf
-----	--------------

References

Nasri (2020). On non-central squared copulas. *Statistics and Probability Letters*.

Examples

```
pncs(c(0.5,0.8),"ncs-clayton", par=c(2,1,2),rotation=270)
```

pplac	<i>Cdf for Plackett copula</i>
-------	--------------------------------

Description

This function computes the distribution function of the Plackett copula with parameter $\text{par} > 0$.

Usage

```
pplac(data, rotation = 0, par)
```

Arguments

data	Matrix (x,y) of size n x 2
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	Copula parameter > 0

Value

cdf	Value of cdf
-----	--------------

Examples

```
pplac(c(0.5,0.8),270,3)
```

```
preparedata          Computes unique values, cdf and pdf
```

Description

This function computes the unique values, cdf and pdf for a series of data.

Usage

```
preparedata(x)
```

Arguments

```
x          Vector
```

Value

```
values      Unique (sorted) values
m           Number of unique values
Fn          Empirical cdf of the unique values
fn          Empirical pdf of the unique values
```

References

B.R. Nasri (2022). Tests of serial dependence for arbitrary distributions

C. Genest, J.G. Neslehova, B.N. Remillard and O. Murphy (2019). Testing for independence in arbitrary distributions.

```
#' @examples x = c(0,0,0,2,3,1,3,1,2,0) out = prepare_data(x)
```

```
rhoplackett          Spearman's rho for Plackett copula
```

Description

Computes the theoretical Spearman's rho for Plackett copula

Usage

```
rhoplackett(cpar, rotation = 0)
```

Arguments

```
cpar          Copula parameter; can be a vector.
rotation      Rotation: 0 (default value), 90, 180, or 270.
```

Value

rho Spearman's rho

References

Remillard (2013). Statistical Methods for Financial Engineering. CRC Press

Examples

```
rhoplackett(3,rotation=90)
```

rncs

Simulation of non-central squared copula

Description

This function computes generates a bivariate sample from a non-central squared copula (ncs) associated with a one-parameter copula with parameter cpar, and parameters a1, a2 >0 .

Usage

```
rncs(n, family, rotation = 0, par)
```

Arguments

n Number of observations

family Copula family: "ncs-gaussian", "ncs-clayton", "ncs-frank", "ncs-gumbel", "ncs-joe", "ncs-plackett".

rotation Rotation: 0 (default value), 90, 180, or 270.

par vector of copula parameter and non-centrality parameter a1,a2 >0

Value

U Observations

References

Nasri (2020). On non-central squared copulas. Statistics and Probability Letters.

Examples

```
rncs(100,"ncs-clayton",par=c(2,1,2))
```

rplac	<i>Generates observations from the Plackett copula</i>
-------	--

Description

This function generates observations from a Plackett copula with parameter $\text{par} > 0$.

Usage

```
rplac(n, rotation = 0, par)
```

Arguments

n	Number of pairs to be generated
rotation	Rotation: 0 (default value), 90, 180, or 270.
par	Copula parameter > 0

Value

U	Matrix of observations
---	------------------------

Examples

```
rplac(10, rotation=90, par=2)
```

singumbel	<i>Simulated data</i>
-----------	-----------------------

Description

Simulated data from a Gumbel copula with parameter 2, Bernoulli margin for X1 and zero-inflated Gaussian margin for X2.

Usage

```
data(singumbel)
```

Format

Data frame of numerical values

Examples

```
data(singumbel)  
plot(singumbel, xlab="X1", ylab="X2")
```

statcvm	<i>Goodness-of-fit statistics</i>
---------	-----------------------------------

Description

Computation of goodness-of-fit statistics (Cramer-von Mises and the Kendall's tau)

Usage

```
statcvm(object)
```

Arguments

object Object of class 'EstBiCop'.

Value

Sn	Cramer-von Mises statistic
Tn	Kendall's statistic
Rn	Spearman's statistic
tauemp	Empirical Kendall's tau
tauth	Kendall's tau of the multilinear theoretical copula
rhoemp	Empirical Spearman's rho
rhoth	Spearman's rho of the multilinear theoretical copula
Y1	Ordered observed values of X1
F1	Empirical cdf of Y1
Y2	Ordered observed values of X2
F2	Empirical cdf of Y2
cpar	Copula parameters
family	Copula family
rotation	Rotation value
n	Sample size

References

Nasri & Remillard (2023). Identifiability and inference for copula-based semiparametric models for random vectors with arbitrary marginal distributions. arXiv 2301.13408.

Examples

```
set.seed(2)
data = matrix(rpois(20,1),ncol=2)
out0 = EstBiCop(data,"gumbel")
out = statcvm(out0)
```

taucop	<i>Kendall's tau for a copula family</i>
--------	--

Description

This function computes Kendall's tau for a copula family

Usage

```
taucop(family_number, cpar, rotation = 0)
```

Arguments

family_number	Integer from 1 to 10
cpar	Copula parameters
rotation	Rotation: 0 (default value), 90, 180, or 270.

Value

tau	Kendall's tau
-----	---------------

Examples

```
taucop(4,2,270) # Gumbel copula
```

tauplackett	<i>Kendall's tau for Plackettfamily</i>
-------------	---

Description

This function computes Kendall's tau for Plackett family using numerical integration

Usage

```
tauplackett(cpar, rotation = 0)
```

Arguments

cpar	Copula parameter >0
rotation	Rotation: 0 (default value), 90, 180, or 270.

Value

tau	Kendall's tau
-----	---------------

tauplackett

23

Examples

`tauplackett(2,270)`

Index

* datasets

 singumbel, [20](#)

AuxFun, [2](#)

AuxFunC, [3](#)

BiEmpCdf, [4](#)

CdfInv, [5](#)

dncs, [5](#)

dplac, [6](#)

EmpCdf, [7](#)

est_options, [10](#)

EstBiCop, [7](#)

EstDep, [9](#)

Finv, [11](#)

fnumber, [11](#)

GofBiCop, [12](#)

hncs, [14](#)

hplac, [15](#)

identifiability, [15](#)

pncs, [16](#)

pplac, [17](#)

preparedata, [18](#)

rhoplackett, [18](#)

rncs, [19](#)

rplac, [20](#)

singumbel, [20](#)

statcvm, [21](#)

taucop, [22](#)

tauplackett, [22](#)